Heat transfer and water migration in unsaturated freezing soils *(within the active layer)*

Instructor:
Zhen Liu

Final project
CE5890

Presented By:
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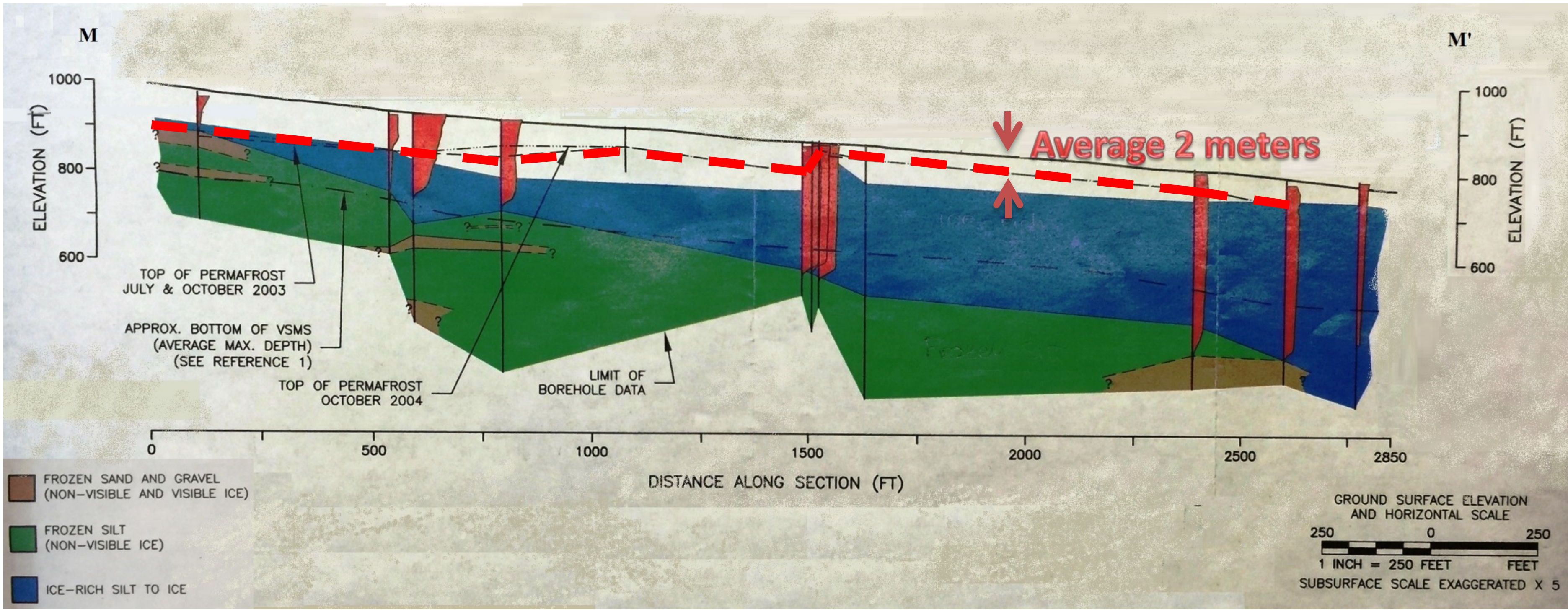


M
M'
Average 2 meters
1000
800
600
ELEVATION (FT)
TOP OF PERMAFROST
JULY & OCTOBER 2003
APPROX. BOTTOM OF VSMS
(AVERAGE MAX. DEPTH)
(SEE REFERENCE 1)
TOP OF PERMAFROST
OCTOBER 2004
LIMIT OF
BOREHOLE DATA
0
500
1000
1500
2000
2500
2850
DISTANCE ALONG SECTION (FT)
FROZEN SAND AND GRAVEL
(NON-VISIBLE AND VISIBLE ICE)
FROZEN SILT
(NON-VISIBLE ICE)
ICE-RICH SILT TO ICE
GROUND SURFACE ELEVATION
AND HORIZONTAL SCALE
250
0
250
1 INCH = 250 FEET
FEET
SUBSURFACE SCALE EXAGGERATED X 5

Governing Equations

Fourier's equation for heat conduction

(The **convective term** is not considered because of the comparatively smaller rate of water migration to that of heat conduction, and also because of the phase change of water)

$$\rho c \frac{\partial T}{\partial t} - \frac{\partial}{\partial x}\left(\lambda \frac{\partial T}{\partial x}\right) = \rho_w \, L \, \frac{\rho_i}{\rho_w} \frac{\partial \theta_i}{\partial t}$$

ρ = soil density
ρ_w = density of water
C = gravimetric specific heat capacity of freezing soil
T = temperature (in kelvin)
t = time
λ = thermal conductivity
L = latent heat of fusion of water
ρ_i = density of ice
θ_i = volumetric ice content
x = is the coordinate in the direction of gravity

Mixed-type Richard's equation (including the term incorporating ice formation)

$$\frac{\partial \theta_w}{\partial t} + \frac{\rho_i}{\rho_w} \frac{\partial \theta_i}{\partial t} + \frac{\partial}{\partial x}\left[\frac{k}{\rho_w g} \left(\frac{\partial \Psi}{\partial x} - 1\right)\right] = 0$$

θ_w = volumetric unfrozen water content
K = hydraulic conductivity
g = gravitational acceleration
Ψ = soil suction

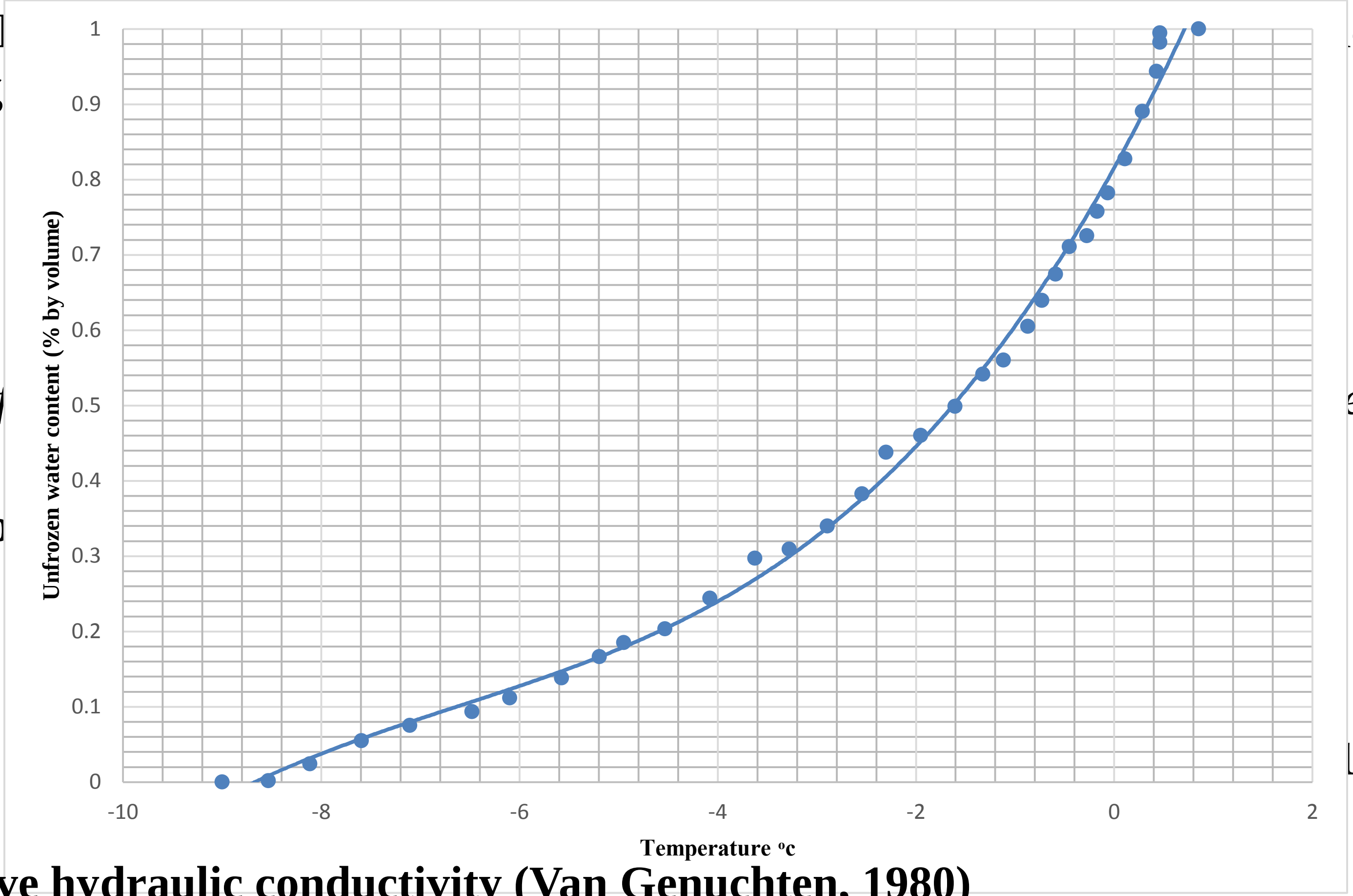
Other Equations

General ... ction for freezing

T_0 = ... essure

Relatio

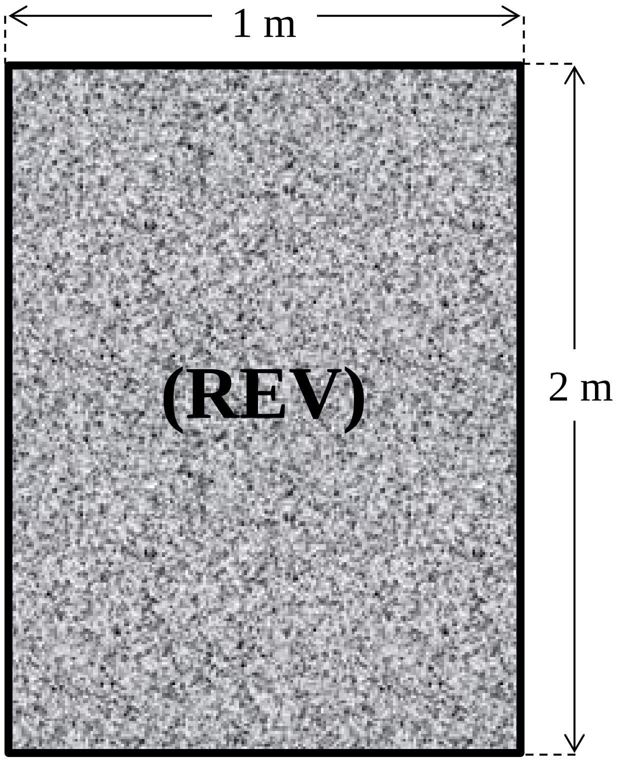
θ_w ... 157

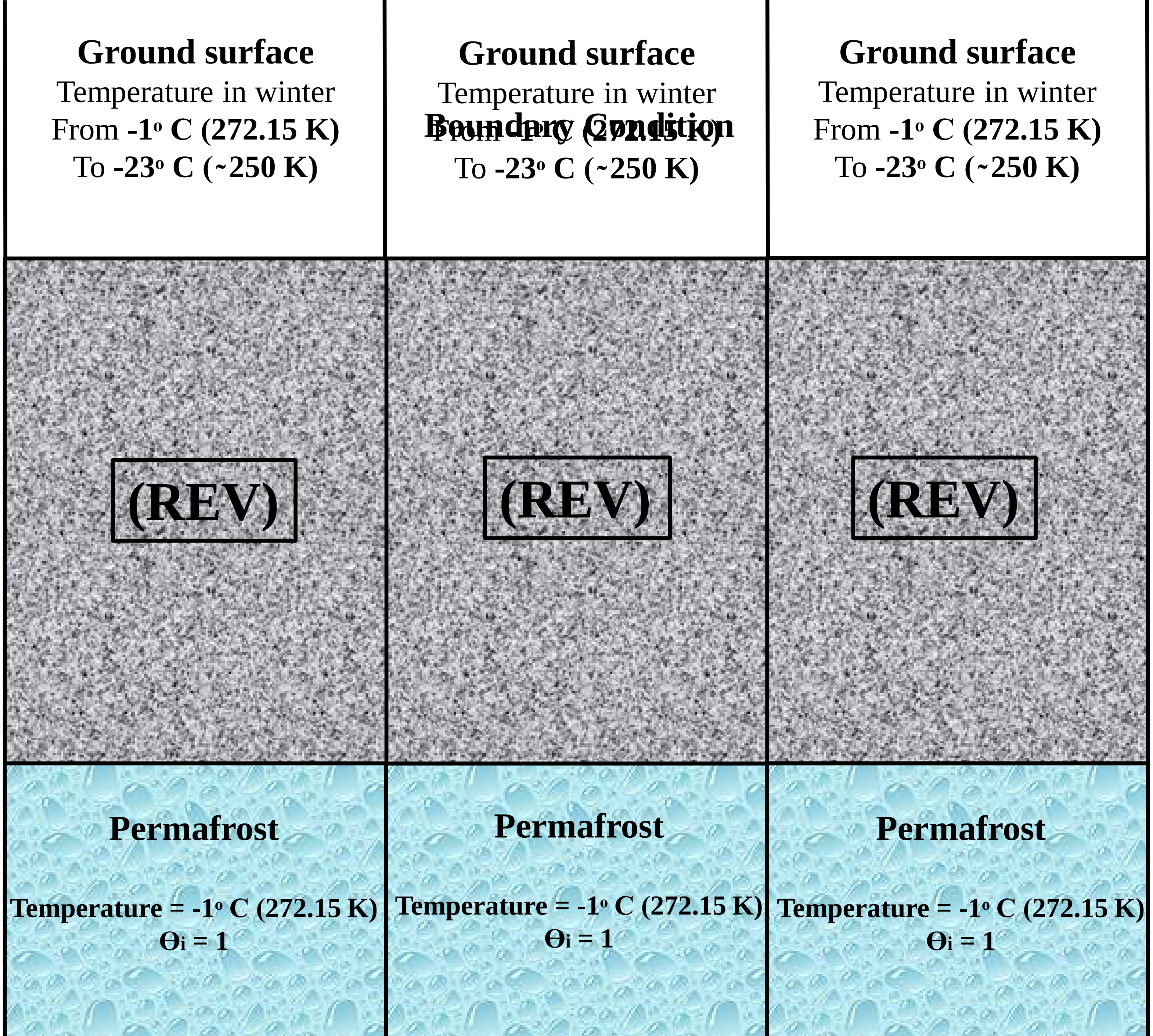


Relative hydraulic conductivity (Van Genuchten, 1980)

$$K_r(\theta_w) = \theta_w{}^2 \left[1 - \left(1 - \theta_w{}^{1/m}\right)^m\right] \qquad m = 1 - 2/n \;, \qquad (0 < m < 1; n > 2)$$

Representative **E**lementary **V**olume (**REV**)



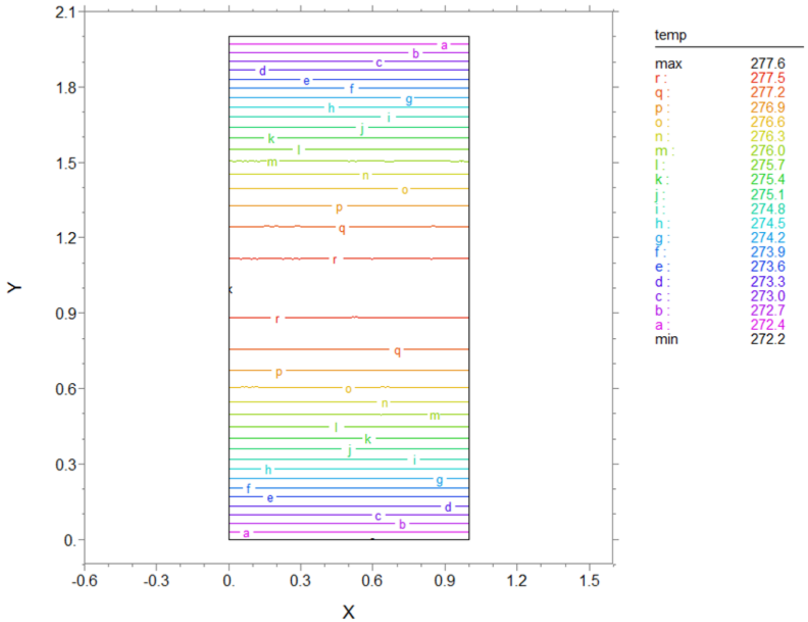


Ground surface
Temperature in winter
From -1° C (272.15 K)
To -23° C (~250 K)
Ground surface
Temperature in winter
Boundary Condition
From -1° C (272.15 K)
To -23° C (~250 K)
Ground surface
Temperature in winter
From -1° C (272.15 K)
To -23° C (~250 K)
(REV)
(REV)
(REV)
Permafrost
Temperature = -1° C (272.15 K)
Θi = 1
Permafrost
Temperature = -1° C (272.15 K)
Θi = 1
Permafrost
Temperature = -1° C (272.15 K)
Θi = 1

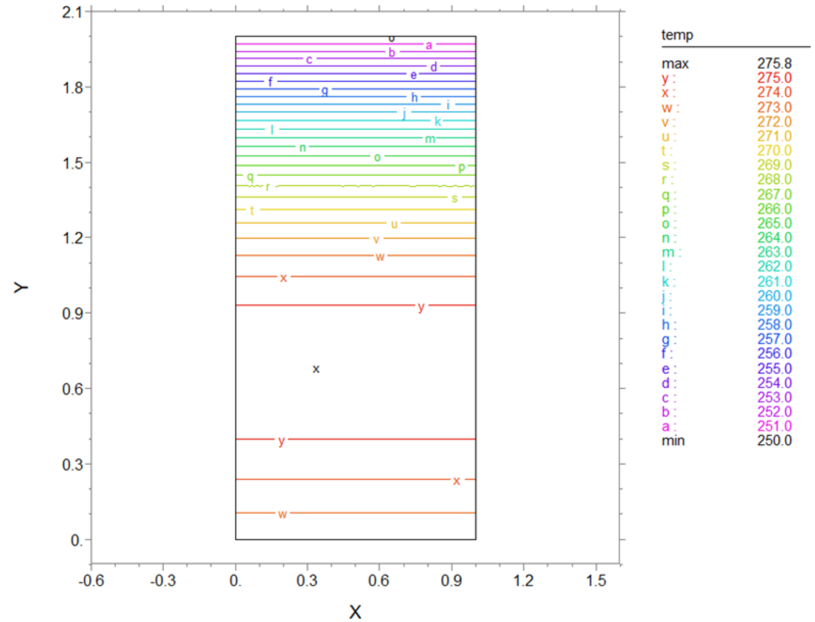
Different Scenarios:

- **Constant K**
  - ✓ Temperature at the ground surface -1^{o}C
  - ✓ Temperature at the ground surface -23^{o}C
- **K as a function of θ_w**
  - ✓ Temperature at the ground surface -1^{o}C
  - ✓ Temperature at the ground surface -23^{o}C

K as a function of θ_w

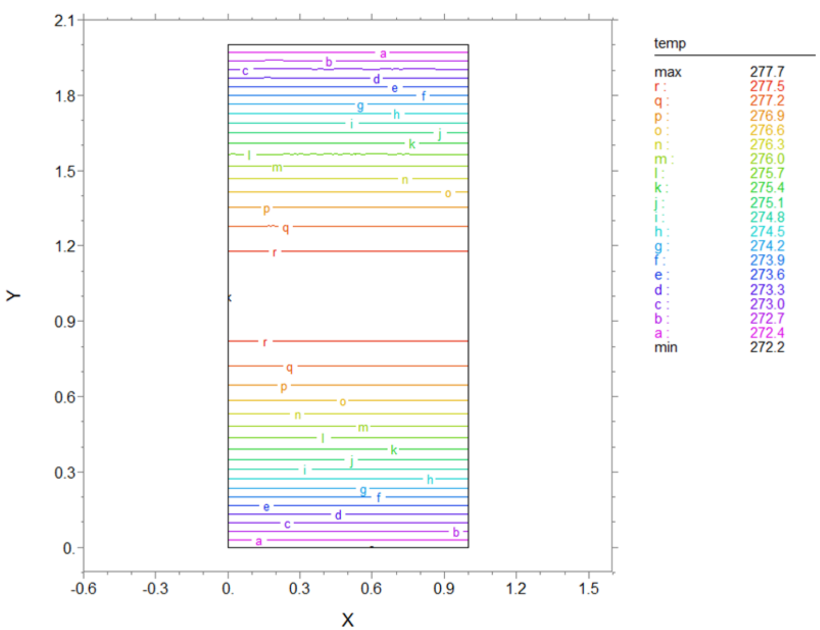


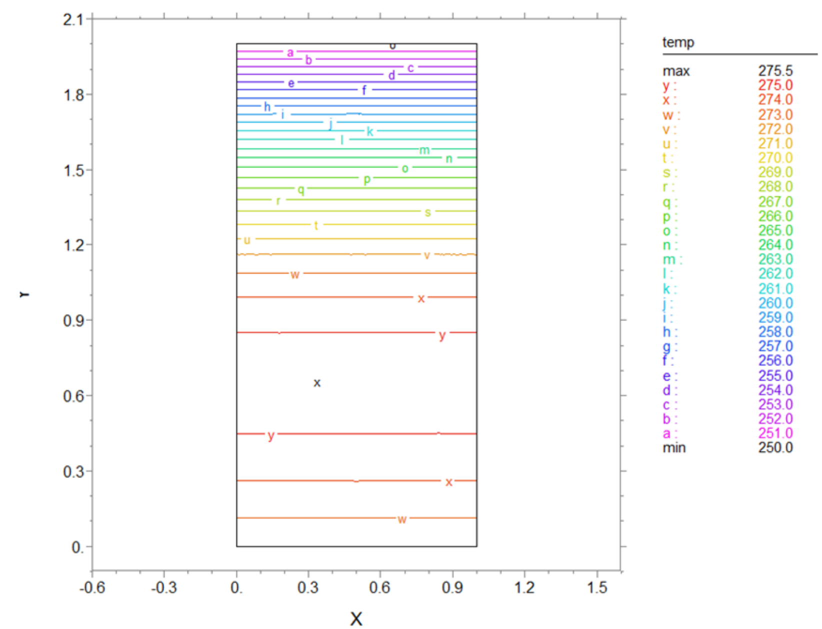
Temperature at the ground surface **-1°C**



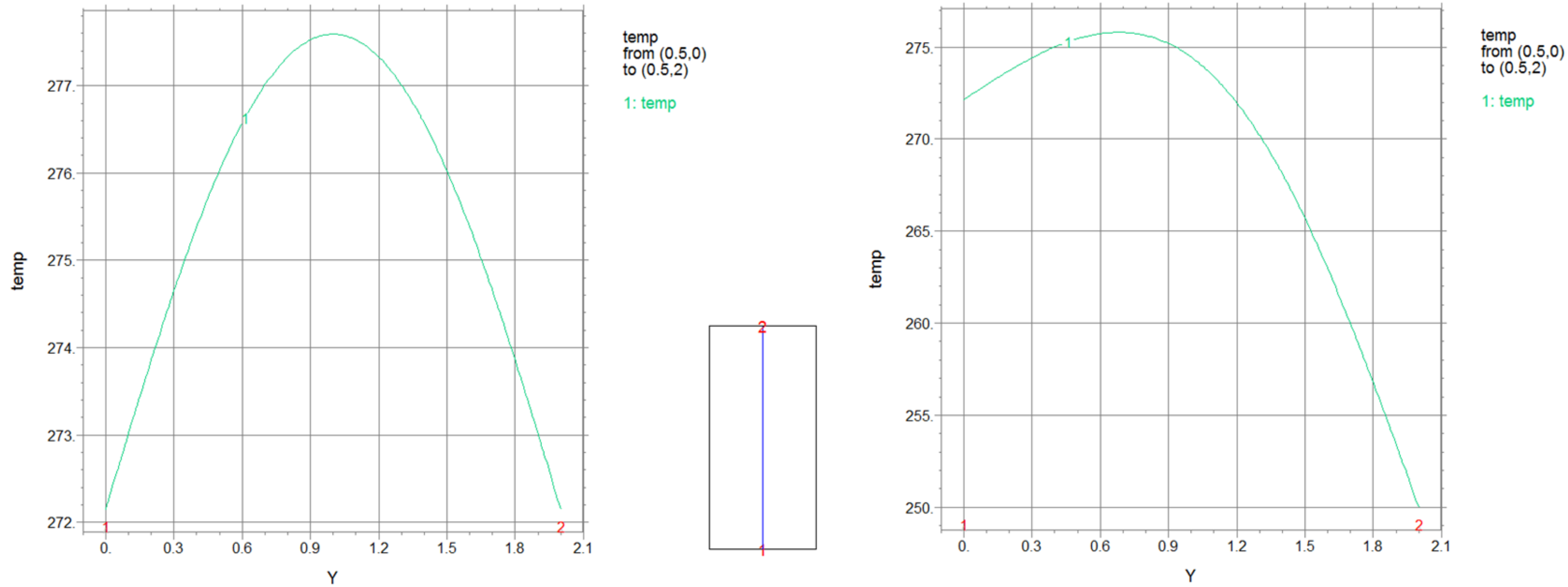
Temperature at the ground surface **-23°C**

Constant K





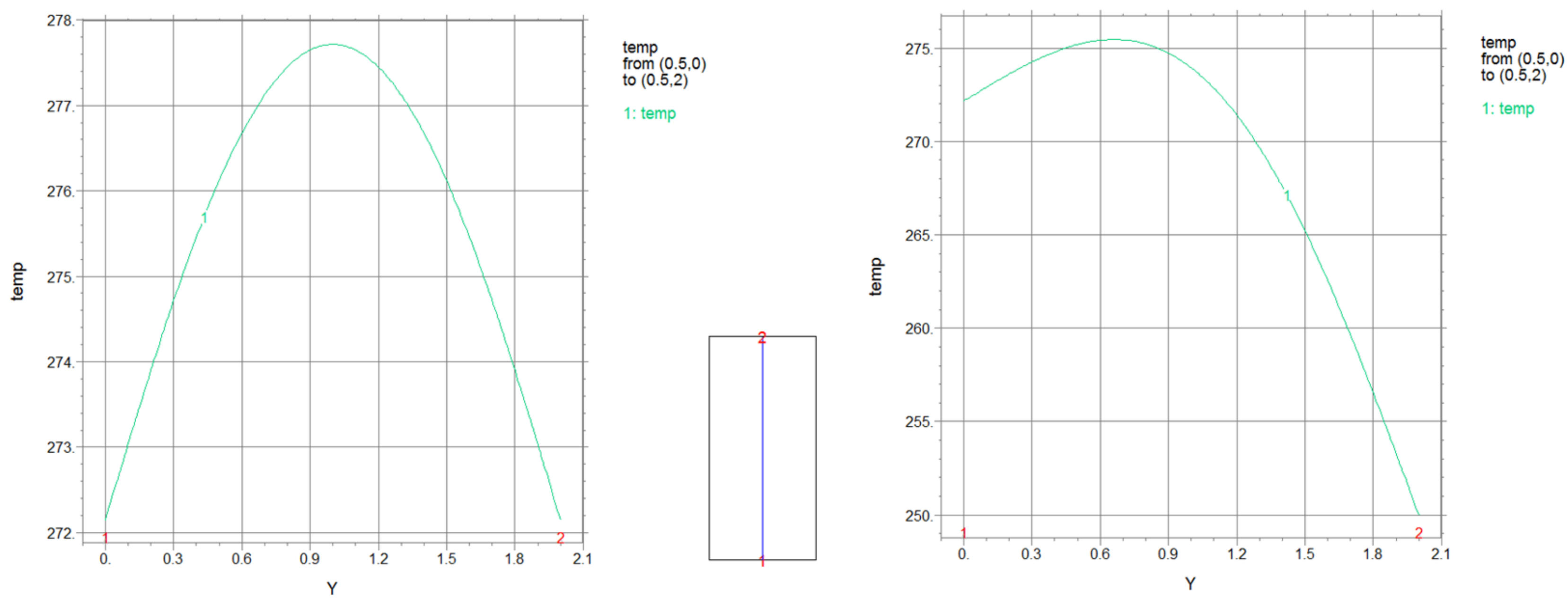
K as a function of θ_w



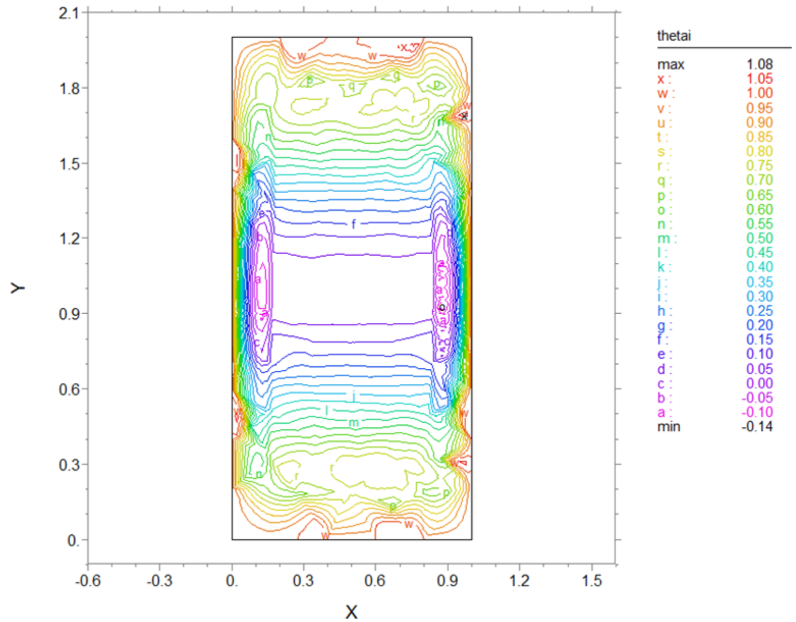
Temperature at the ground surface -1°C

Temperature at the ground surface -23°C

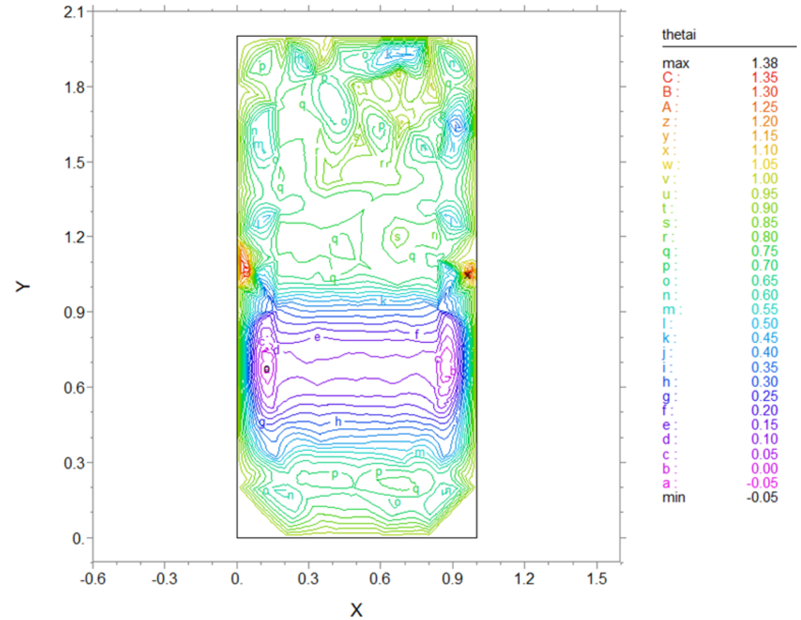
Constant K



K as a function of θ_w

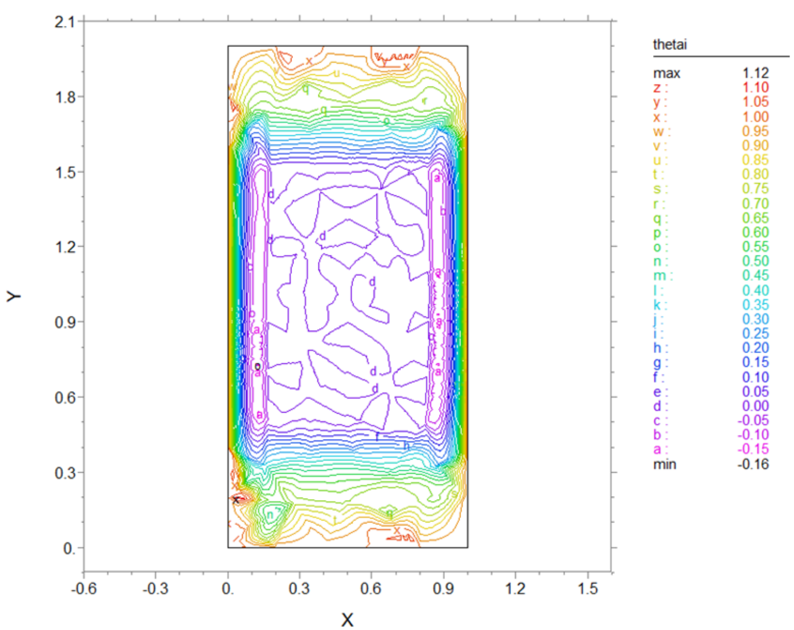


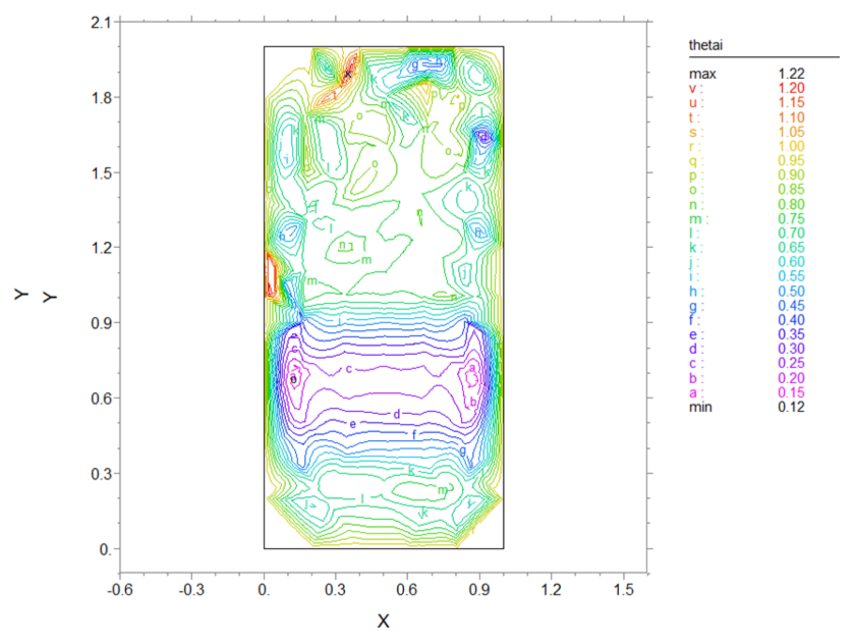
Temperature at the ground surface **-1°C**



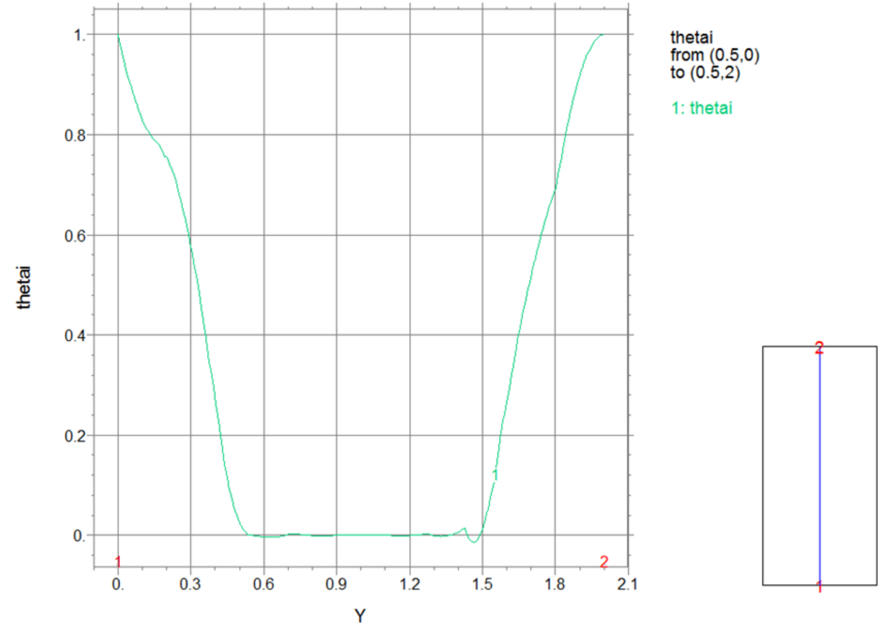
Temperature at the ground surface **-23°C**

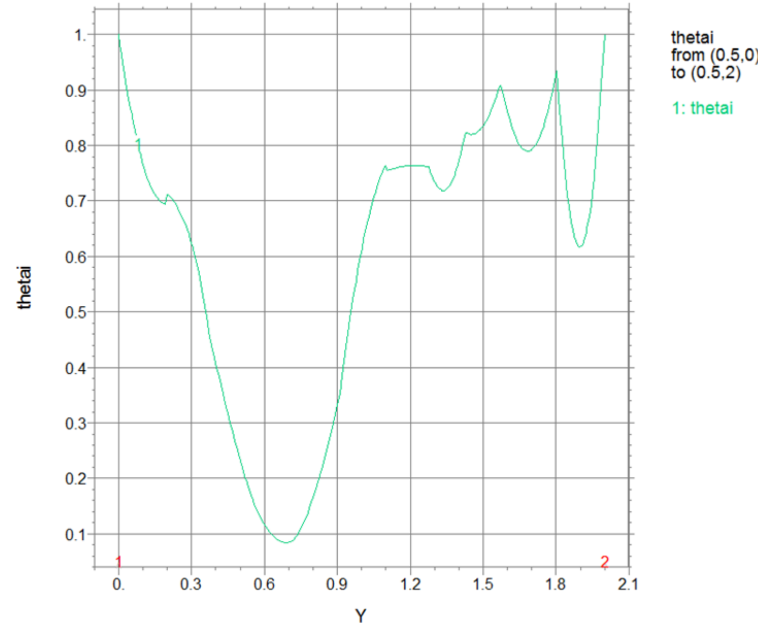
Constant K





K as a function of θ_w

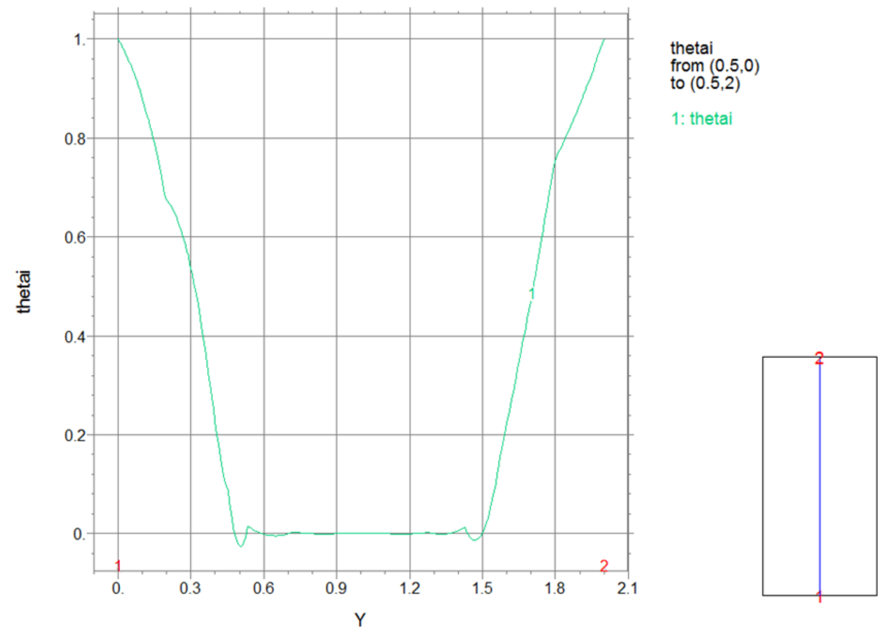


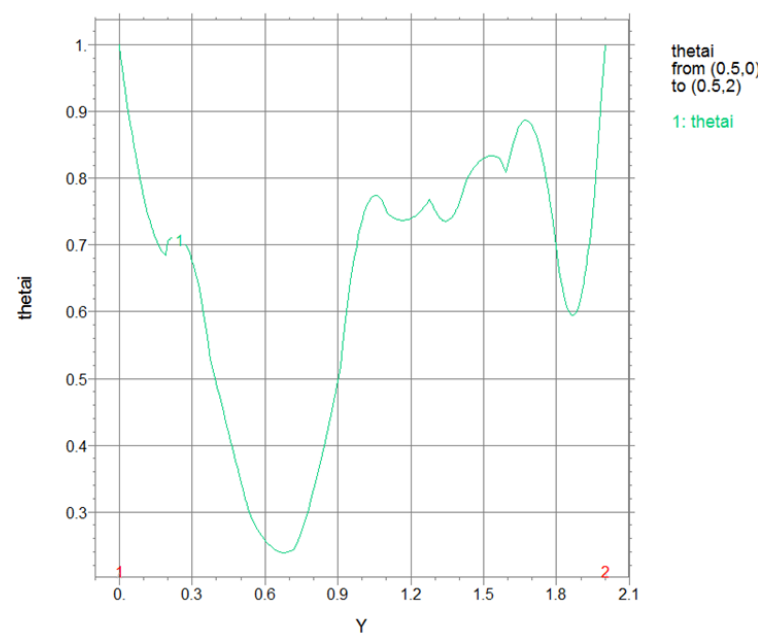


Temperature at the ground surface **-1°C**

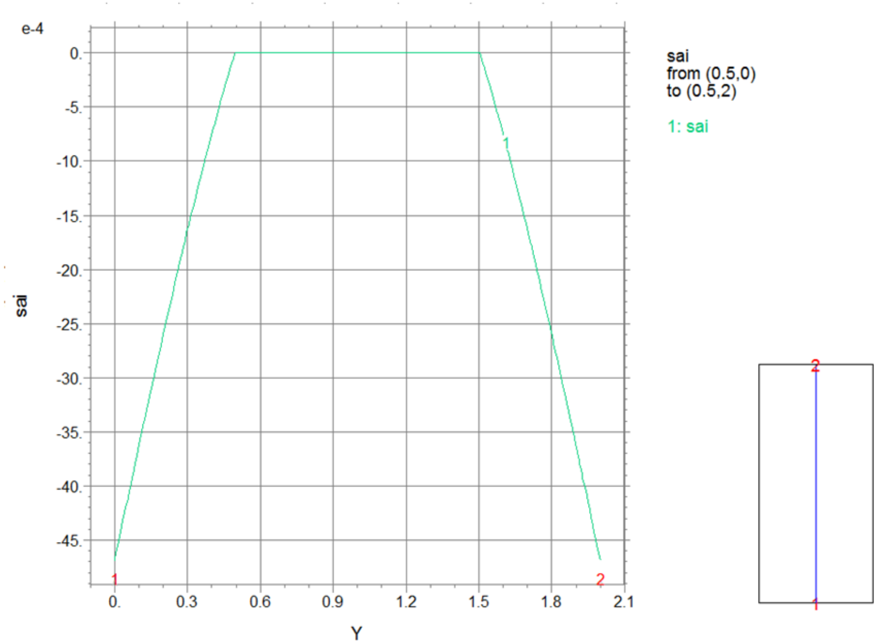
Temperature at the ground surface **-23°C**

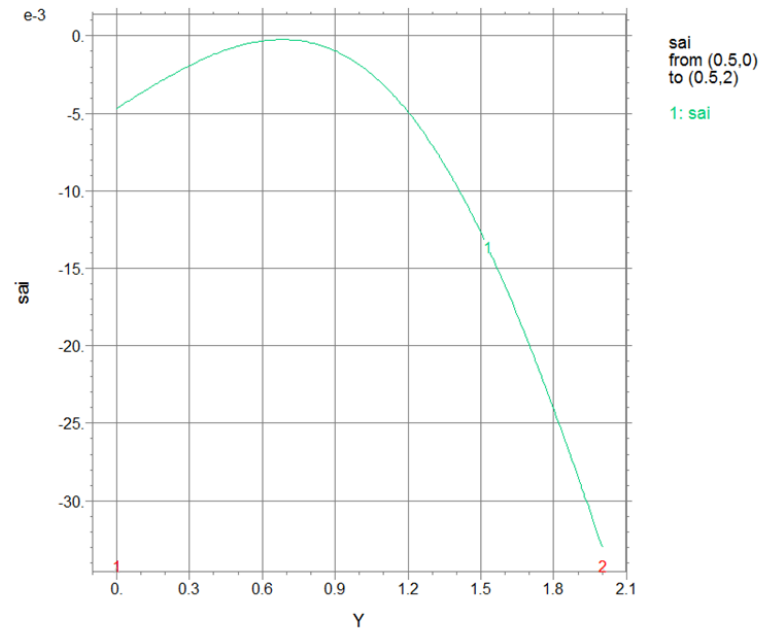
Constant K





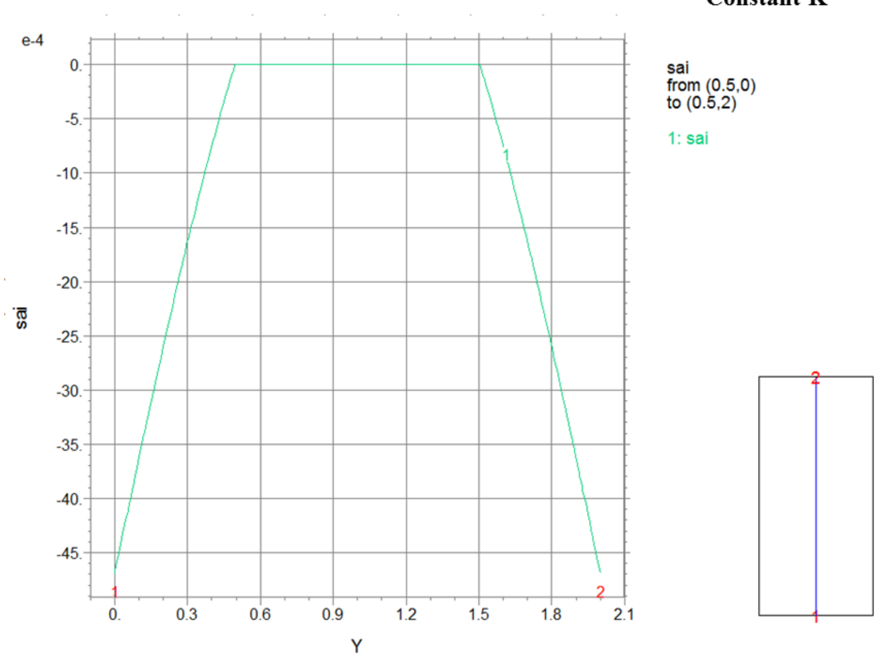
K as a function of θ_w

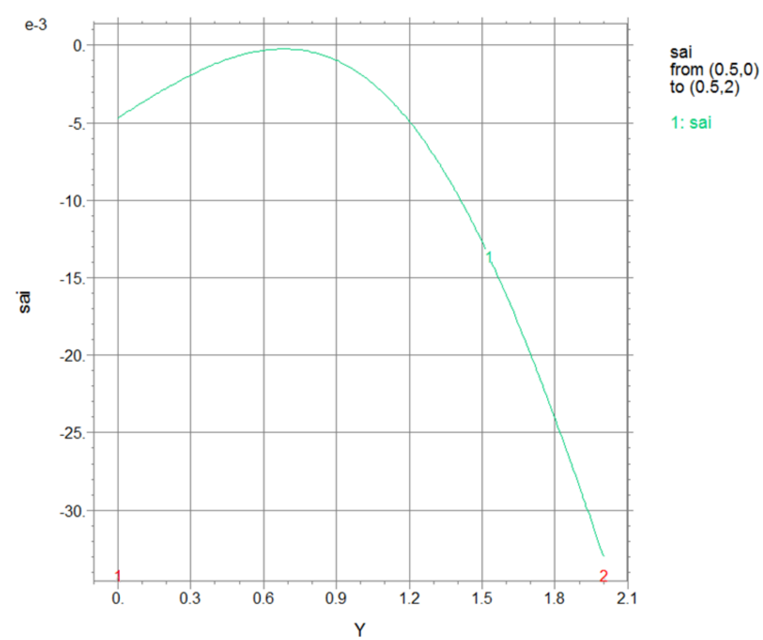




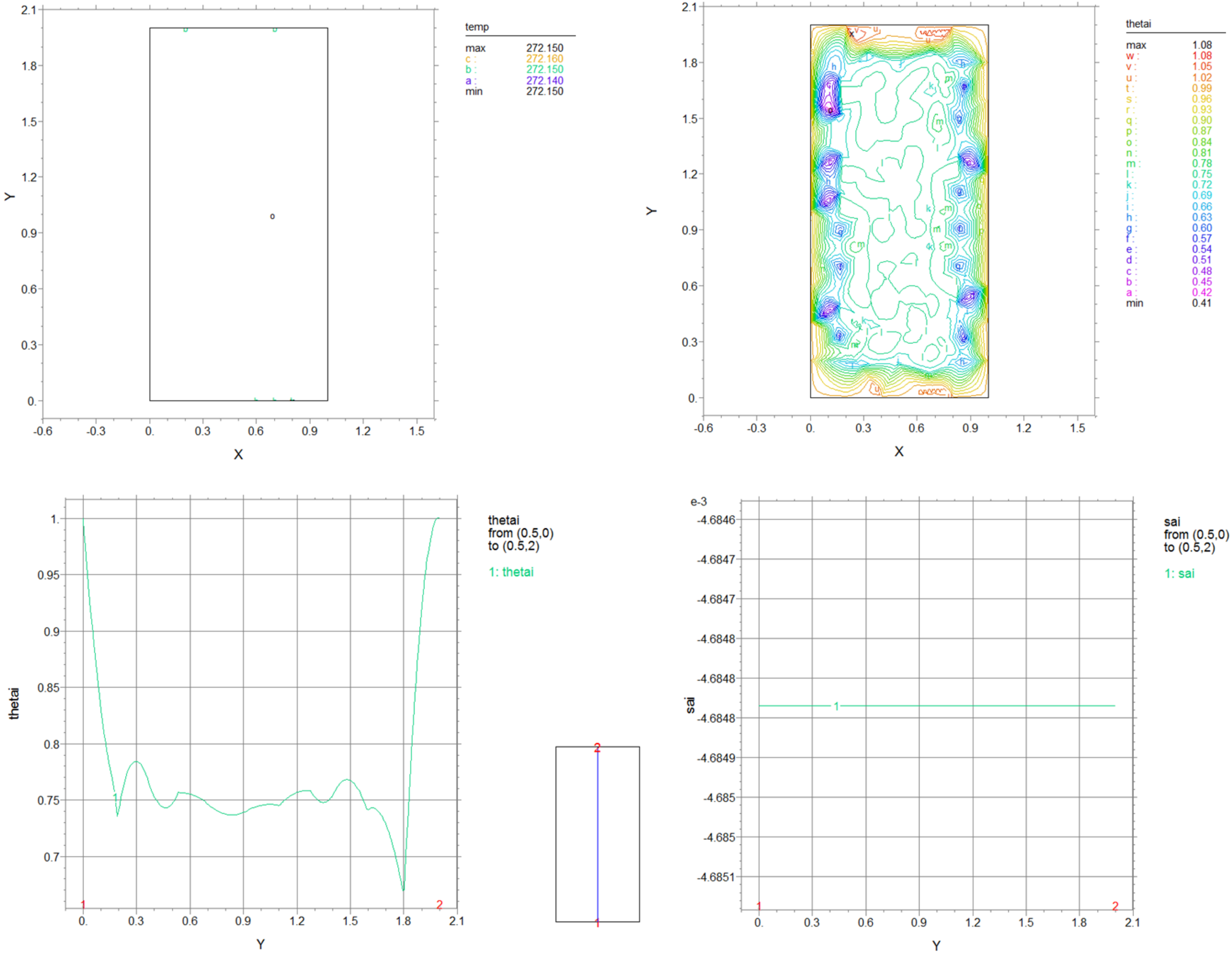
Temperature at the ground surface **-1°C**

Constant K





K as a function of θ_w



Conclusion:

The results of our simulations for Low temperatures at ground surface **(-1^{o}C)** are in a very good agreement with what expected

When the temperature at the ground surface is too cold **(-23^{o}C)**, the results seem to at least have the expected trend but instead of having a smooth curve, its fluctuating

Possible explanation of non smooth results:

Software does not allow us to refining mesh so large mesh might be the reason of fluctuations in the results

Fourier's equation for heat conduction

(The **convective term** is not considered because of the comparatively smaller rate of water migration to that of heat conduction, and also because of the phase change of water)

$$\rho c \frac{\partial T}{\partial t} - \frac{\partial}{\partial x}\left(\lambda \frac{\partial T}{\partial x}\right) = \rho_w L \frac{\rho_i}{\rho_w}\frac{\partial \theta_i}{\partial t}$$

ρ = *soil density*
ρ_w = *density of water*
C = *gravimetric specific heat capacity of freezing soil*
T = *temperature (in kelvin)*
t = *time*
λ = *thermal conductivity*
L = *latent heat of fusion of water*
ρ_i = *density of ice*
θ_i = *volumetric ice content*
x = *is the coordinate in the direction of gravity*

Mixed-type Richard's equation (including the term incorporating ice formation)

$$\frac{\partial \theta_w}{\partial t} + \frac{\rho_i}{\rho_w}\frac{\partial \theta_i}{\partial t} + \frac{\partial}{\partial x}\left[\frac{k}{\rho_w g}\left(\frac{\partial \Psi}{\partial x} - 1\right)\right] = 0$$

θ_w = *volumetric unfrozen water content*

K = *hydraulic conductivity*

g = *gravitational acceleration*

Ψ = *soil suction*

Other Equations

Generalized Clapeyron Equation (integrated and written in the form of soil suction for freezing soils):

$$\Psi = -\rho_w L \ln \frac{T}{T_0}$$

T_0 = freezing point of bulk water (in kelvin) under a standard atmospheric pressure

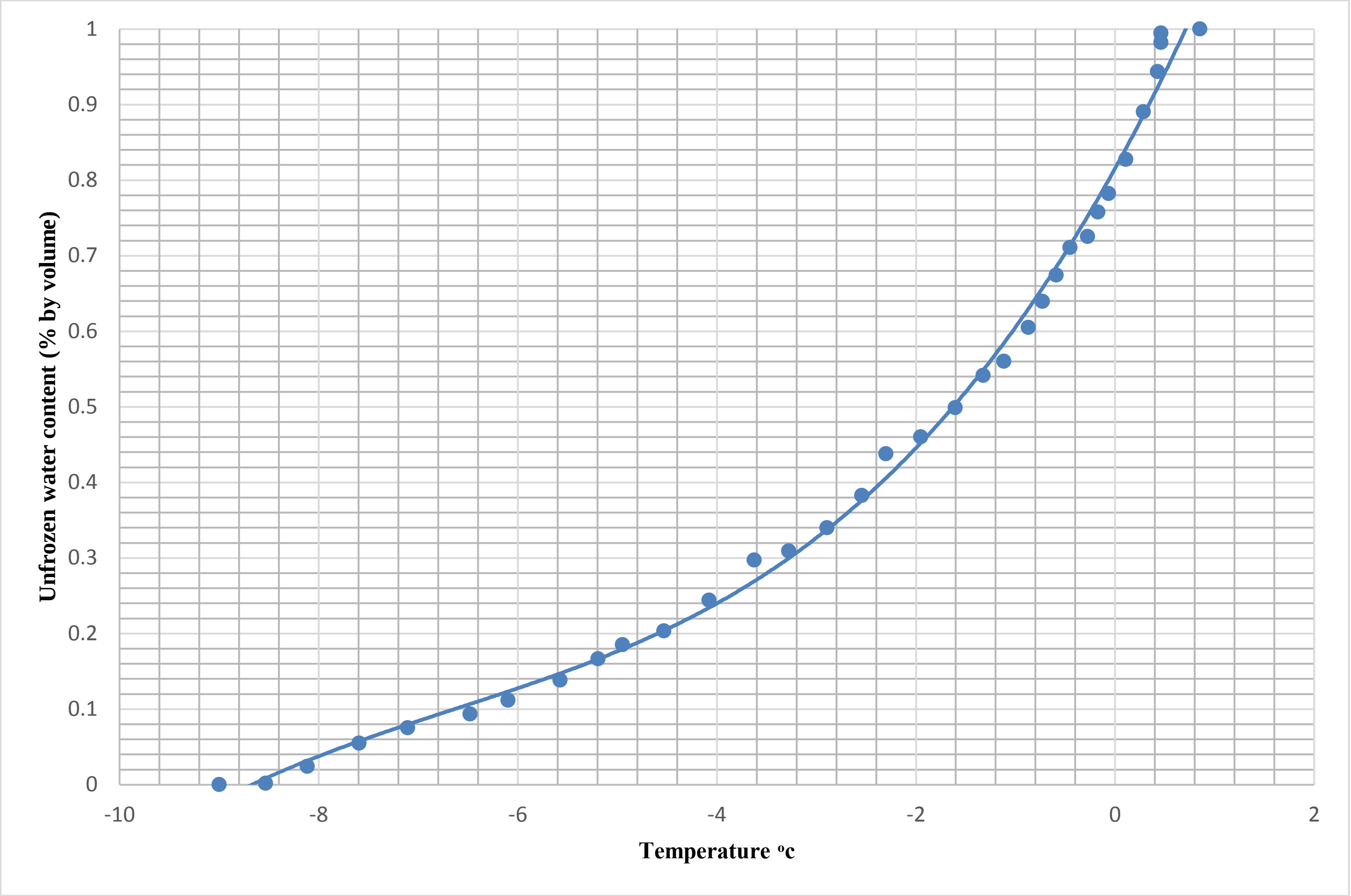
Relationships between the temperature and the water content

$$\theta_w = \theta_w(T)$$

$$\theta_w = 0.0015(T - 273.15)^3 + 0.0294(T - 273.15)^2 + 0.238 \text{ x } + 0.8157$$

Relative hydraulic conductivity (Van Genuchten, 1980)

$$K_r(\theta_w) = \theta_w{}^2 \left[1 - \left(1 - \theta_w{}^{1/m}\right)^m\right] \qquad m = 1 - 2/n \;, \qquad (0 < m < 1; n > 2)$$



Unfrozen water content (% by volume)
1
0.9
0.8
0.7
0.6
0.5
0.4
0.3
0.2
0.1
0
-10
-8
-6
-4
-2
0
2
Temperature ºc